647510 **BSCmaMJ201x-1C** **Seat No :________**

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2024
MATH: .Matrices and Coordinate Geometry (Major-1)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any ONE out of TWO **(05)**

(1) Define: Multiplication of matrices.
Prove that multiplication of matrices is distributive.

(2) Define: Hermitian and skew-Hermitian matrices.
Show that every square matrix is uniquely expressible as a sum of a Hermitian and a skew-Hermitian matrices.

Que.1(B) Answer any ONE out of TWO **(05)**

(1) Define: Adjoint of a square matrix.
$Prove\ that\quad A\cdot(adjA)=|A|\cdot I=(adjA)\cdot A$
$where\ A\ is\ a\ square\ matrix\ of\ order\ n.$

(2) Define: Orthogonal matrix.
$Find\ the\ values\ of\ l, m\ and\ n\ if\ \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix} is\ orthogonal.$

Que.2(A) Answer any ONE out of TWO **(05)**

(1) Define: Rank of a matrix. If the matrices A & B are conformable for multiplication, then prove that $\rho(AB)\le\min\{\rho(A),\rho(B)\}$.

(2) Define: Normal form of a matrix. Find the rank of $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ by representing it into the normal form.

Que.2(B) Answer any ONE out of TWO **(05)**

(1) Define: Echelon form. Find the rank of $\begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$ by converting it into Echelon form.

(2) Prove that the row rank of a matrix is equal to the rank of it.

Que.3(A) Answer any ONE out of TWO **(05)**

(1) State and prove the Cayley-Hamilton theorem.

(2) Define: Eigen Value and Eigen Vector of a square matrix. Find the eigen values and eigen vectors of the matrix $A=\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$.

Que.3(B) Answer any ONE out of TWO **(05)**

(1) By using the Cayley-Hamilton theorem, find the inverse of the matrix $A=\begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix}$.

(2) State and prove the necessary and sufficient conditions for a system of simultaneous linear equations $AX=B$ to be consistent.

Que.4(A) **Answer any ONE out of TWO** **(05)**

(1) State the relation between Cartesian and Polar coordinates. Convert $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ into polar form (where a is constant).

(2) Derive the general equation of a circle in polar form.

Que.4(B) **Answer any ONE out of TWO** **(05)**

(1) Derive general equation of a straight line in polar form. Also derive the condition for two lines in polar form to be perpendicular.

(2) Derive the relation between Cartesian and Spherical coordinate of a point in $\mathbb{R}^3$.

Que.5(A) **Answer any ONE out of TWO** **(05)**

(1) Define:

(i) Sub Matrix (ii) Determinant

(iii) Idempotent Matrix (iv) Nilpotent Matrix

(v) Involutory Matrix.

(2) Find the rank of the matrix $A = \begin{pmatrix} 3 & 2 & 0 & -1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{pmatrix}$.

Que.5(B) **Answer any ONE out of TWO** **(05)**

(1) Solve the following system.

$$8x + 6y + 5z + 2w = 21$$
$$3x + 3y + 2z + w = 10$$
$$4x + 2y + 3z + w = 8$$
$$3x + 5y + z + w = 15$$

(2) Derive the equation of a straight line in $\boldsymbol{p - \alpha}$ form.

****